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# Top Quark Production at TeV Energies as a Potential SUSY Detector \*

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## Abstract

We consider the process of top-antitop production from electron-positron annihilation, for c. m. energies in the few TeV regime, in the MSSM theoretical framework. We show that, at the one loop level, the slopes of a number of observable quantities in an energy region around 3 TeV are only dependent on  $\tan \beta$ . Under optimal experimental conditions, a combined measurement of slopes might identify  $\tan \beta$  values in a range  $\tan \beta < 2$ ,  $\tan \beta > 20$  with acceptable precision.

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## I. INTRODUCTION.

The existence of large electroweak Sudakov logarithms [1] in four-fermion processes at the one-loop level, for c.m. energies in the TeV range [2,3], and the subsequent efforts for providing a full resummation of the relevant terms [4,5], have been the subject of recent theoretical motivated discussions and effort, whose final elaboration is still in progress within the theoretical SM framework. More recently, the extension of this kind of analysis at one loop within the MSSM, for final SM massless fermion [6] and top quark pair production [7], has been also accomplished. Finally, very recently, an analysis of the role of the so called “ $\theta$ -independent” and “ $\theta$ -dependent” Sudakov effects at one loop has been presented [8]. These two terms are, briefly, those for which a clean resummation prescription exists, and those for which this conclusion does not seem to be valid [5]. In the t’Hooft  $\xi = 1$  gauge, as shown in [8], the latter terms are subleading linear logarithms in the c.m. energy  $\sqrt{q^2}$ , originated by a  $\theta$ -dependent ( $\theta$  is the c.m. scattering angle) component of the  $W$  (and, to a much smaller extent,  $Z$ ) boxes.

One of the conclusions of [7] is that the process of top pairs production from electron-positron annihilation at c.m. energies in the few TeV range, i.e. those that will be explored by the future CERN CLIC collider [9], is particularly lucky for what concerns the validity of a SM calculation. Owing to the weak isospin characterization of the final state, in fact, the non resummable  $\theta$ -dependent contribution turns out to be quite small [8], so that only the resummable  $\theta$ -independent component survives in the SM, making a clean theoretical estimate of the various observables already (in principle) available. Note that similar conclusions would not apply, for instance, for bottom production, due to its opposite isospin value.

The fact that virtual SM effects are fully under control raises the interest of considering the role of possible virtual SUSY effects of Sudakov origin in top production at CLIC energies. On very general grounds, we shall say that to be considered as potentially “interesting”, such effects should be of an “intelligent” type. By this we mean that they should be visible, and therefore “sufficiently large” with respect to the experimental accuracy at the computable one-loop level. On the other hand they should also be “sufficiently small” at the same level, not to spoil its presumed perturbative validity (given the fact that a two loop calculation of SUSY virtual effects seems to us, at least at the moment, unrealistic for this process.)

It was shown in [7] that the leading Sudakov SUSY effect at one loop is only of linear logarithmic kind (in the SM, also quadratic logarithms appear), and  $\theta$ -independent. It is only produced by final vertices, and contains a component of “massless quark” kind and one of “massive top” Yukawa origin that strongly depends on  $\tan\beta$ . Its numerical effect can vary from a few percent to more than ten percent, strongly depending on  $\tan\beta$ , being apparently still sensitive to relatively large  $\tan\beta$  values  $> 10$ . In [7], it was suggested that this fact might be used to try to fix the value of  $\tan\beta$  from a combined analysis of the value of several observables at fixed energy, e.g. around the proposed CLIC “optimal” value  $\sqrt{q^2} = 3 \text{ TeV}$ . The conclusion of that Reference was that a deeper investigation of this possibility would have followed.

The aim of this short paper is precisely that of performing the aforementioned investigation. Our proposal will be that of considering, rather than measurements at fixed energies,

variations of observables (slopes) with energy around a chosen specially interesting (e.g. 3 TeV) energy value. We shall show that the only unknown quantities in the coefficients of the various slopes are functions of  $\tan\beta$  alone. All the other MSSM parameters can be incorporated asymptotically into terms that either vanish or remain constant, thus disappearing in the slope. We shall then consider a reasonable experimental setup and try to conclude that it might be possible to identify  $\tan\beta$  with “decent” precision (e.g. to better than a relative fifty percent) up to relatively large  $\tan\beta$  values. This should be compared and combined with other interesting recently proposed  $\tan\beta$  detection techniques [10].

Technically speaking, this short paper is organized as follows. In Section II we shall present our one-loop derivation of the SUSY slopes in the few TeV region. In Section III, we shall propose our “data simulation” and  $\tan\beta$  identification, and present a few general concluding remarks in the final Section IV.

## II. SUSY SUDAKOV LOGARITHMS IN THE TEV REGION

A first treatment of top production at one loop in the MSSM at TeV energies has been already given [7], and all the relevant formulae for observables can be found there in Section IV. For the specific purposes of this paper, we shall rewrite them here in a form where the SM component of all observables has been considered as a given perfectly known theoretical input. This statement requires the following precise explanation. At few TeV energies, it has been argued very recently [8] that the one-loop theoretical description in the SM is often in trouble, due to the presence of large and opposite “ $\theta$ -independent” and “ $\theta$ -dependent” Sudakov terms. The possible way out would be represented by a resummation of the separately large logarithms. Unfortunately, a clean resummation prescription at the moment only seems to exist for the so called “ $\theta$ -independent” terms, and not for the “ $\theta$ -dependent” ones [5], so that a completely satisfactory set of theoretical predictions in the few TeV (CLIC) energy range must still be provided.

A remarkable exception to this negative statement is represented by top pair production. Here, for reasons that are simply connected with the top weak isospin assignment [8], the  $\theta$ -dependent terms are strongly depressed at one loop, so that their resummation does not seem necessary. A partial resummation of the separate  $\theta$ -independent terms would thus guarantee a fully satisfactory theoretical prediction for the SM component of the process, leaving the SUSY component as the only quantity to be investigated.

Following this attitude, we shall thus rewrite all the relevant formulae of [7] indicating with the “SM” label the (supposedly perfectly known) SM component. For our purposes, we shall need the asymptotic Sudakov expansion of the various quantities. The latter contains as the leading term a  $\theta$ -independent, linear logarithm. This is, as we said in the Introduction, the sum of a “massless” and a “massive” component, whose origin is due to the vertex diagrams shown in Fig.(1). In the limit when the c.m. energy  $\sqrt{q^2}$  becomes very large, they produce the overall leading logarithmic SUSY Sudakov terms listed in the following equations for photon or  $Z$  exchanges:

$$\begin{aligned} \Gamma_\mu^\gamma(\text{MSSM, massive}) \rightarrow & \frac{e\alpha}{12\pi M_W^2 s_W^2} \ln \frac{q^2}{M^2} \{ m_t^2 (1 + \cot^2 \beta) [(\gamma_\mu P_L) + 2(\gamma_\mu P_R)] \\ & + m_b^2 (1 + \tan^2 \beta) (\gamma_\mu P_L) \} \end{aligned} \quad (2.1)$$

$$\begin{aligned}\Gamma_\mu^Z(\text{MSSM, massive}) \rightarrow & \frac{e\alpha}{48\pi M_W^2 s_W^3 c_W} \ln \frac{q^2}{M^2} \{(3 - 4s_W^2)m_t^2(1 + \cot^2 \beta)(\gamma_\mu P_L) \\ & - 8s_W^2 m_t^2(1 + \cot^2 \beta)(\gamma_\mu P_R) + (3 - 4s_W^2)m_b^2(1 + \tan^2 \beta)(\gamma_\mu P_L)\} \quad (2.2)\end{aligned}$$

$$\Gamma_\mu^\gamma(\text{MSSM, massless}) \rightarrow \frac{e\alpha}{12\pi s_W^2 c_W^2} \ln \frac{q^2}{M^2} \left\{ \left( 3 - \frac{26}{9}s_W^2 \right) P_L + \frac{16}{9}s_W^2 P_R \right\} \quad (2.3)$$

$$\Gamma_\mu^Z(\text{MSSM, massless}) \rightarrow \frac{e\alpha}{32\pi s_W^3 c_W^3} \ln \frac{q^2}{M^2} \left\{ \left( 3 - \frac{62}{9}s_W^2 + \frac{104}{27}s_W^4 \right) P_L - \frac{64}{27}s_W^4 P_R \right\} \quad (2.4)$$

where  $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ .

One sees that, in the leading term, the only SUSY parameters that appear are  $\tan \beta$  and an overall common unknown “SUSY scale”  $M$  which we only assumed to be “reasonably” smaller than the energy value  $\sqrt{q^2} = 3$  TeV in which we are interested in this work. Of course, this assumption might be wrong and heavier SUSY masses might turn out to be produced. In that case, our “asymptotic” expansions would still be valid, obviously in a suitably larger energy range.

Starting from Eqs.(2.1),(2.2), it is a straightforward task to derive the leading SUSY Sudakov contributions to the various observables. We write here, in the previously discussed spirit, the expressions that will be relevant for our purposes, considering, for simplicity, first a set of observables where the final top quark helicity is not measured and secondly a set of 4 observables where it is. The chosen quantities are  $\sigma_t$  (the cross section for top pair production),  $A_{FB,t}$  and  $A_{LR,t}$  (the forward-backward and longitudinal polarization asymmetries) and  $A_t$  (the polarized forward-backward asymmetry) in the first set. In the second one, we have considered  $H_t$  (the averaged top helicity),  $H_{FB,t}$  (its forward-backward asymmetry),  $H_t^{LR}$  (the averaged polarized top helicity) and  $H_{t,FB}^{LR}$  (its forward-backward asymmetry), and all these quantities are defined in detail in the Appendix B of [7]. For the chosen observables we obtain the following asymptotic expansions:

$$\sigma_t = \sigma_t^{SM} \left\{ 1 + \frac{\alpha}{4\pi} \left\{ (4.44 N + 11.09) \ln \frac{q^2}{\mu^2} - 10.09 \ln \frac{q^2}{M^2} \right\} + F_{\sigma_t}(\tan \beta) \ln \frac{q^2}{M^2} \right\} \quad (2.5)$$

$$F_{\sigma_t}(t) = \frac{\alpha}{4\pi} (-29 t^{-2} - 0.0084 t^2 - 14),$$

$$A_{FB,t} = A_{FB,t}^{SM} + \frac{\alpha}{4\pi} \left\{ (0.22 N + 1.29) \ln \frac{q^2}{\mu^2} - 0.23 \ln \frac{q^2}{M^2} \right\} + F_{A_{FB,t}}(\tan \beta) \ln \frac{q^2}{M^2}. \quad (2.6)$$

$$F_{A_{FB,t}}(t) = \frac{\alpha}{4\pi} (1.2 t^{-2} - 0.00082 t^2 + 0.62),$$

$$A_{LR,t} = A_{LR,t}^{SM} + \frac{\alpha}{4\pi} \left\{ (1.03 N + 5.95) \ln \frac{q^2}{\mu^2} - 4.03 \ln \frac{q^2}{M^2} \right\} + F_{A_{LR,t}}(\tan \beta) \ln \frac{q^2}{M^2} \quad (2.7)$$

$$F_{A_{LR,t}}(t) = \frac{\alpha}{4\pi} (7.7 t^{-2} - 0.0051 t^2 + 3.8),$$

$$A_t = A_t^{SM} + \frac{\alpha}{4\pi} \left\{ (0.91 N + 5.25) \ln \frac{q^2}{\mu^2} - 3.20 \ln \frac{q^2}{M^2} \right\} + F_{A_t}(\tan \beta) \ln \frac{q^2}{M'^2}, \quad (2.8)$$

$$F_{A_t}(t) = \frac{\alpha}{4\pi} (7.5 t^{-2} - 0.0049 t^2 + 3.7),$$

$$H_t = H_t^{SM} + \frac{\alpha}{4\pi} \left\{ (-1.21 N - 7.00) \ln \frac{q^2}{\mu^2} + 4.27 \ln \frac{q^2}{M^2} \right\} + F_{H_t}(\tan \beta) \ln \frac{q^2}{M'^2}, \quad (2.9)$$

$$F_{H_t}(t) = \frac{\alpha}{4\pi} (-9.9 t^{-2} + 0.0066 t^2 - 5),$$

$$H_{t,FB} = H_{t,FB}^{SM} + \frac{\alpha}{4\pi} \left\{ (-0.77 N - 4.46) \ln \frac{q^2}{\mu^2} + 3.02 \ln \frac{q^2}{M^2} \right\} + F_{H_{t,FB}}(\tan \beta) \ln \frac{q^2}{M'^2}, \quad (2.10)$$

$$F_{H_{t,FB}}(t) = \frac{\alpha}{4\pi} (-5.7 t^{-2} + 0.0038 t^2 - 2.9),$$

$$H_t^{LR} = H_t^{LR,SM} + \frac{\alpha}{4\pi} \left\{ (-0.30 N - 1.71) \ln \frac{q^2}{\mu^2} + 0.31 \ln \frac{q^2}{M^2} \right\} + F_{H_t^{LR}}(\tan \beta) \ln \frac{q^2}{M'^2}, \quad (2.11)$$

$$F_{H_t^{LR}}(t) = \frac{\alpha}{4\pi} (-1.6 t^{-2} + 0.0011 t^2 - 0.82),$$

$$H_{t,FB}^{LR} = H_{t,FB}^{LR,SM} + \frac{\alpha}{4\pi} \left\{ (0.) \ln \frac{q^2}{\mu^2} + (0.) \ln \frac{q^2}{M^2} \right\} + F_{H_{t,FB}^{LR}}(\tan \beta) \ln \frac{q^2}{M'^2}, \quad (2.12)$$

$$F_{H_{t,FB}^{LR}}(t) = \text{constant}$$

In the previous equations, we have also listed in the first term, for sake of completeness, the SUSY asymptotic linear logarithm of RG origin. This contributes a universal term of self-energy origin, where no SUSY parameters (except a SUSY scale) appear. In our procedure we shall add this RG logarithm to that of SM origin, and consider it as an uninteresting part of the “Non SUSY Sudakov” structure, that will be treated as a known contribution in our “slopes-based” procedure. The second term is the massless Sudakov term (i.e. the one that would have been obtained for  $u, c$  pair production) and the third one contains the massive Sudakov term as a function of  $\tan \beta$ .

One notices, as stressed in [7], a strong  $\tan \beta$  dependence in some observable (particularly  $\sigma_t$ ) that might possibly be exploited to perform a determination of this parameter. With this purpose, we have examined the above possibility with some caution, in a way that shall now illustrate.

Clearly, if the logarithmic term were the only relevant one in the SUSY component of the observables, a determination of  $\tan \beta$  might proceed in principle via a fit of the various observables at a fixed chosen energy. Quite generally, in an asymptotic expansion like the one that we are assuming, there will be extra non leading contributions, in particular constant terms and terms that vanish (at least as  $1/q^2$ ) asymptotically. We assumed consistently within our philosophy that the latter ones can be safely neglected and concentrated our attention on possible constant quantities. Our approach was that of computing exactly the contributions to the various observables from the considered SUSY vertices, and to try to fit the numerical results with an expression of the kind  $(F_1 \cot^2 \beta + F_2 \tan^2 \beta + F_3) \log q^2 + G$ , as discussed in the forthcoming Section.

### III. NUMERICAL VALIDITY OF THE ASYMPTOTIC EXPANSION AND A POSSIBLE PROCEDURE FOR THE DETERMINATION OF $\tan \beta$

To check the validity of the Sudakov asymptotic expansion we have computed the relevant complete one loop SUSY effects in the MSSM by evaluating without approximations all the diagrams that are not vanishing in the large energy limit, considering a (ideal) situation in which at least some of the SUSY parameters have been measured with reasonable precision while other ones remain possibly still undetermined.

The large set of free parameters of the MSSM has been chosen for a first approach as follows: we have assumed the Grand Unification relation  $M_1 = \frac{5}{3} \sin^2 \theta_W M_2$  between the  $U(1)$  and  $SU(2)$  gaugino parameters; we have fixed the mass of the lightest chargino at 200 GeV, the  $\mu$  parameter at 500 GeV and the mass of the CP odd neutral Higgs boson at 300 GeV; we have considered a mixed sfermion sector characterized by a sfermion mass scale  $M_S = 300$  GeV.

For each observable we have attempted a simple parametrization of the SUSY effect of the form

$$F \log \frac{q^2}{M_Z^2} + G \quad (3.1)$$

where  $F$  and  $G$  depend on the observable and in principle also on all the model parameters. In fact, we know that if we are in an energy range where the Sudakov expansion is holding than the coefficient  $F$  depends on  $\tan \beta$  only and admits the simple parametrization

$$F \equiv F(\tan \beta) = F_1 \cot^2 \beta + F_2 \tan^2 \beta + F_3 \quad (3.2)$$

with  $F_1$ ,  $F_2$  and  $F_3$  in agreement with Eqs. (2.5-2.11).

In Fig.(2) we plot the percentual relative SUSY effect in  $\sigma_t$  computed with the above mentioned set of MSSM parameters and by choosing  $\tan \beta = 2.0$ . On the logarithmic scale it is easy to recognize the asymptotic linear logarithmic behaviour that sets in at energies beyond the threshold  $\sqrt{q^2} \simeq 3$  TeV. A similar behaviour can be seen in all the other observables.

We then have repeated the analysis for several values of  $\tan \beta$  and have tried to determine the dependence of  $F$  and  $G$  on that parameter. The results are shown in Fig.(3). Circles and diamonds are the data points that have been obtained by a fit performed under the conservative cut  $\sqrt{q^2} > 3$  TeV. The solid and dashed lines have been obtained by fitting the data with the functional form Eq. (3.2). The match is quite good and the crucial point is that  $F_1$ ,  $F_2$  and  $F_3$  agree with the analytical Sudakov expansion to a few percent. As such, they are actually known numerical constants independent on other model parameters like gaugino mass parameters or other SUSY scales. In fact, our choice for the set of MSSM parameters is “reasonable” lacking more detailed experimental information and it permits to check if the typical energy at which the Sudakov expansion starts to be reliable is in the CLIC reach. If some MSSM parameter is varied, for instance assuming a heavier lightest chargino, then our conclusions are still valid asymptotically. Although we did not perform an exhaustive investigation of the full parameter case, in several sensible cases  $\sqrt{q^2} > 3$  TeV appears to be a safe threshold for the expansion.

Is it possible to exploit this simplification in the high energy limit to determine the value of  $\tan \beta$ ? To address this question, we define the relative effect on the observable  $\mathcal{O}_n$  as the ratio

$$\epsilon_n(q^2) = \frac{\mathcal{O}_n(q^2) - \mathcal{O}_n^{SM}(q^2)}{\mathcal{O}_n^{SM}(q^2)} = F_n \log \frac{q^2}{M_Z^2} + G_n \quad (3.3)$$

and denote by  $\sigma_n(q^2)$  the experimental error on  $\epsilon_n(q^2)$ , assuming  $\mathcal{O}_n^{SM}$  perfectly known.

We then suppose that a set of  $N$  independent measurements is available at c.m. energies  $\sqrt{q_1^2}, \sqrt{q_2^2}, \dots, \sqrt{q_N^2}$ . Differences with respect to the measurement at lowest energy

$$\delta_{n,i} = \epsilon_n(q_i^2) - \epsilon_n(q_1^2) \quad (3.4)$$

do not contain the constant  $G_n$  (and the SUSY mass scales  $M, M'$  hidden in it) and provide direct access to  $\tan \beta$  through  $F$ . In fact, for each set of explicit measurements  $\{\delta_{n,i}\}$ , the optimal value of  $\tan \beta$  is determined by minimizing the  $\chi^2$  sum

$$\chi^2 = \sum_{i=1}^N \sum_{n=1}^{N_O} \frac{(F_n \log \frac{q_{i+1}^2}{q_1^2} - \delta_{n,i})^2}{4\sigma_{n,i}^2} \quad (3.5)$$

where  $\delta_{n,i} \equiv \delta_n(q_i^2)$  and  $\sigma_{n,i} \equiv \sigma_n(q_i^2)$ . We make the usual assumption that  $\delta_{n,i}$  is a normal Gaussian random variable distributed around the value

$$F_n(\tan \beta^*) \log \frac{q_{i+1}^2}{q_1^2} \quad (3.6)$$

with standard deviation  $2\sigma_{n,i}$ . Hence,  $\beta^*$  is the unknown true value. After linearization around  $\tan \beta = \tan \beta^*$ , minimization of  $\chi^2$  provides the best estimate of  $\tan \beta$  that is also a Gaussian random variable. Its mean is of course  $\tan \beta^*$  and the standard deviation  $\delta \tan \beta$  is given by the condition  $\Delta \chi^2 = 1$

$$\delta \tan \beta = 2 \left( \sum_{n,i} \left( \frac{F'_n(\tan \beta^*) \log \frac{q_{i+1}^2}{q_1^2}}{\sigma_{n,i}} \right)^2 \right)^{-1/2} \quad (3.7)$$

If we simplify the discussion by assuming  $\sigma_{n,i} \equiv \sigma$ , this formula reduces to

$$\delta \tan \beta = 2\sigma \left( \sum_n F'_n(\tan \beta^*)^2 \right)^{-1/2} \left( \sum_i \log^2 \frac{q_{i+1}^2}{q_1^2} \right)^{-1/2} \quad (3.8)$$

The function

$$\tau(\tan \beta) = \left( \sum_n F'_n(\tan \beta)^2 \right)^{-1/2} \quad (3.9)$$

measures the dependence of the slope of SUSY effects on  $\tan \beta$ . It is shown in Fig.(4) for three possible choices: (i) only  $\sigma_t$ , (ii) the four non - helicity observables  $\sigma_t, A_{FB,t}, A_{LR,t}$

and  $A_t$ , (iii) the non-helicity observables and the three helicity observables  $H_t$ ,  $H_{FB,t}$  and  $H_t^{LR}$ .

In the best case (iii), it is strongly peaked around  $\tan \beta = 8$  and the combination of the various observables, especially  $\sigma_t$ ,  $A_{LR,t}$ ,  $A_t$  and  $H_t$  (the ones with larger  $\cot^2 \beta$  coefficient) is crucial to keep the function  $\tau(\tan \beta)$  as small as possible.

To understand the consequences of the shape of  $\tau$ , we plot in Fig.(5) the relative error  $\delta \tan \beta / \tan \beta$  computed under the optimistic assumption of a relative accuracy (absolute for  $\sigma_t$ ) equal to 1% for all the seven observables. The three curves correspond to the assumption that independent measurements for each observable are available at  $N = 5, 10$  or  $20$  equally spaced c.m. energies between  $2$  TeV and  $6$  TeV. Of course, different curves associated to pairs  $(N, \sigma)$  actually depend only on the combination  $\sigma/\sqrt{N}$ . We also show horizontal dashed lines corresponding to relative errors equal to  $1$  and  $0.5$ . As one can see in the Figure, values in the range

$$\tan \beta < 2, \quad \tan \beta > 20 \quad (3.10)$$

can be detected with  $N = 10$  c.m. energy values with a relative error smaller than 50%, that we consider qualitatively as a “decent” accuracy. Obviously, if a higher experimental precision (e.g. a few permille in  $\sigma_t$ ) were achievable, the same result could be obtained with a smaller number ( $N \simeq 3$ ) of independent energy measurements.

#### IV. CONCLUSIONS

We have discussed in this paper the possibility of determining the crucial MSSM parameter  $\tan \beta$  via measurements of the slopes of a number of experimental observables in the process of top-antitop production from electron-positron annihilation in the CLIC regime. We have assumed an “ideal” situation in which some information on the SUSY parameters already exists, and we have fixed some of them to values that appear to us reasonable. Of course, this values might be different than those which (hopefully) will be determined in future measurements. The point remains, though, that independently on this “details” the slopes of the observables will be only dependent on  $\tan \beta$ . Our results show that, in principle, under expected reasonable experimental conditions, it would be possible to derive “decent” informations on  $\tan \beta$  in two ranges, i.e.  $\tan \beta < 2$  and  $\tan \beta > 20$ . This seems to us an interesting possibility, particularly for what concerns the second large  $\tan \beta$  range. In this case, to our knowledge, the realistic possibilities of measuring  $\tan \beta$  are rather restricted and not simple, as exhaustively discussed in a very recent paper [10]. In fact, a measurement of  $\tan \beta$  is practically impossible from chargino or neutralino production when  $\tan \beta > 10$  since the effects depend on  $\cos 2\beta$  that becomes flat for  $\beta \rightarrow \pi/2$ . It could be achieved in the associated productions  $e^+e^- \rightarrow h\tilde{\tau}\tilde{\tau}$  or  $e^+e^- \rightarrow A\bar{b}b$  (with  $h$  and  $A$  being the CP even and odd Higgs bosons), but only for very large  $\tan \beta$  values ( $\sim 50$ ). Our proposal might represent an alternative independent determination, to be possibly combined with other methods, either already proposed or to be suggested in future studies.

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## FIGURES

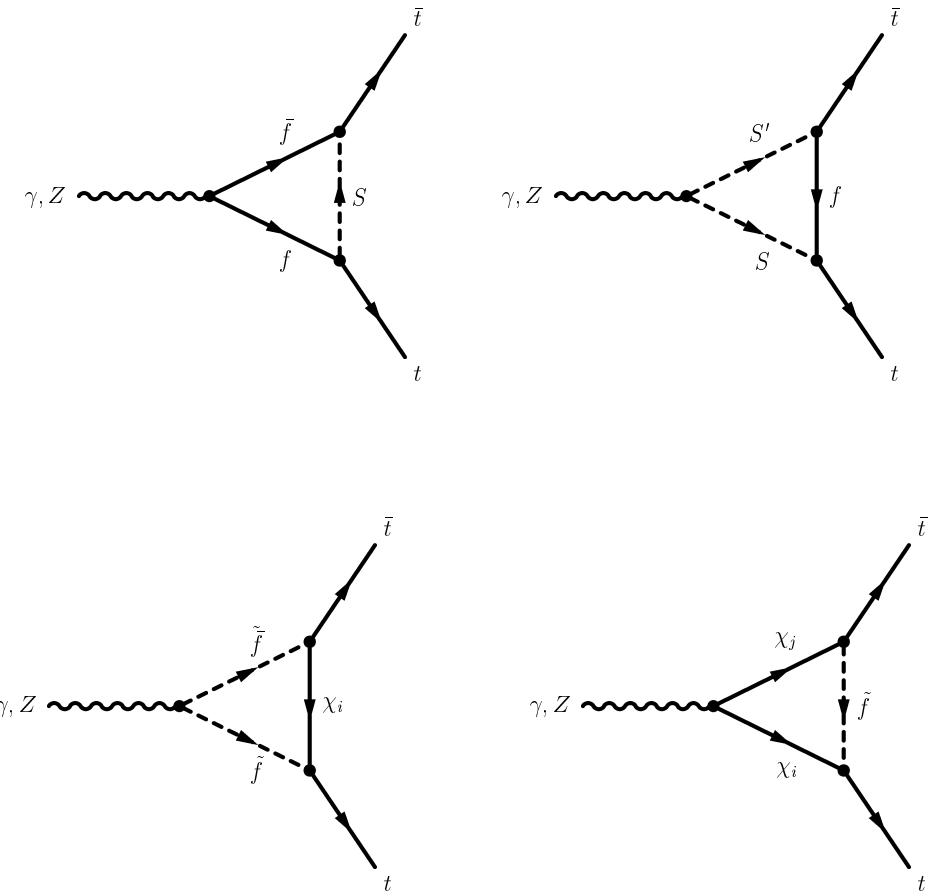


FIG. 1. Triangle diagrams with SUSY Higgs and with SUSY partners contributing to the asymptotic logarithmic behaviour in the energy;  $f$  represent  $t$  or  $b$  quarks;  $S$  represent charged or neutral Higgs bosons  $H^\pm$ ,  $A^0$ ,  $H^0$ ,  $h^0$  or Goldstone  $G^0$ ;  $\tilde{f}$  represent stop or sbottom states;  $\chi$  represent charginos or neutralinos. The arrow corresponds to the momentum flow of the indicated particle.

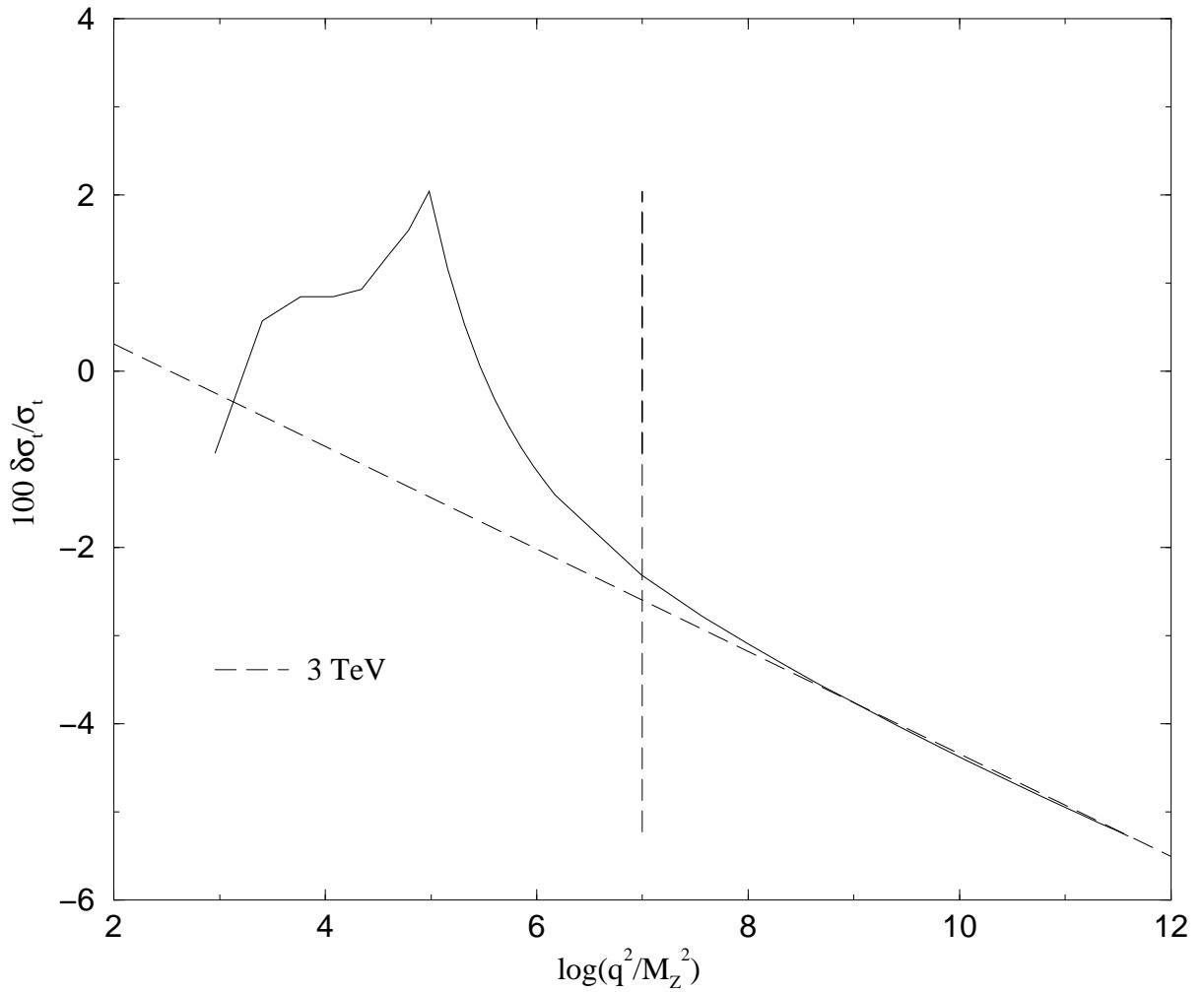


FIG. 2. Plot of  $\sigma_t$ . The set of MSSM parameters is fully described in Section III. The vertical dashed line marks the CLIC 3 TeV energy. The oblique dashed line is a linear logarithmic fit of the curve in the high energy region.

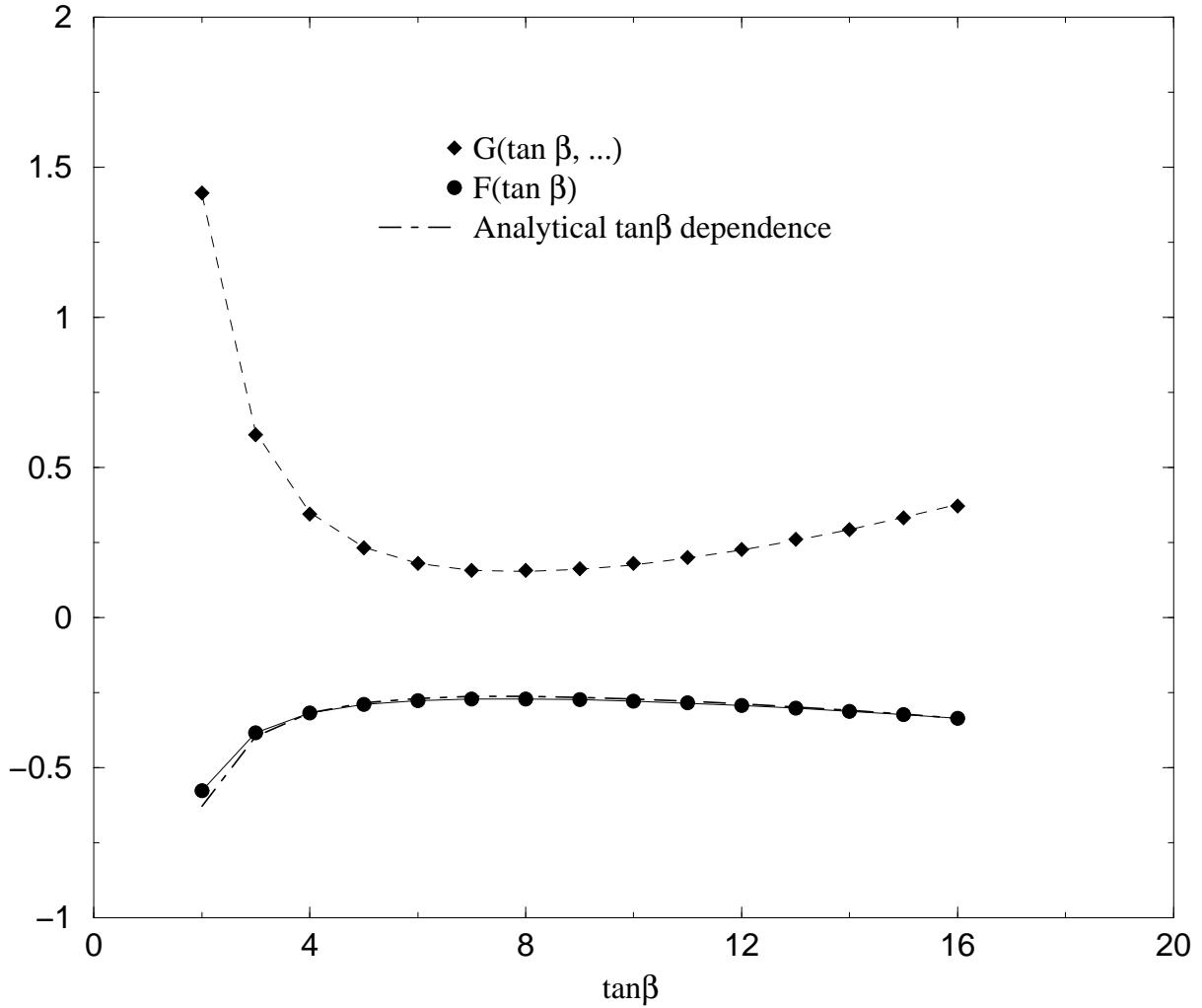


FIG. 3. Logarithmic fit of the SUSY effects in  $\sigma_t$  and  $\tan\beta$  dependence. The data points (circles and diamonds) are the result from a logarithmic fit of the relative SUSY effects in  $\sigma_t$  as in Fig. (2). The solid and dashed lines are nonlinear fits of the functional form  $F_1 \cot^2 \beta + F_2 \tan^2 \beta + F_3$ . As we explained, the results for  $F$  confirm the validity of the analytical Sudakov expansion that is also shown in the Figure (dot-dashed line). It is interesting to remark that the same functional dependence on  $\tan\beta$  seems to work well also for the constant  $G$  (for a given choice of the other MSSM parameters).

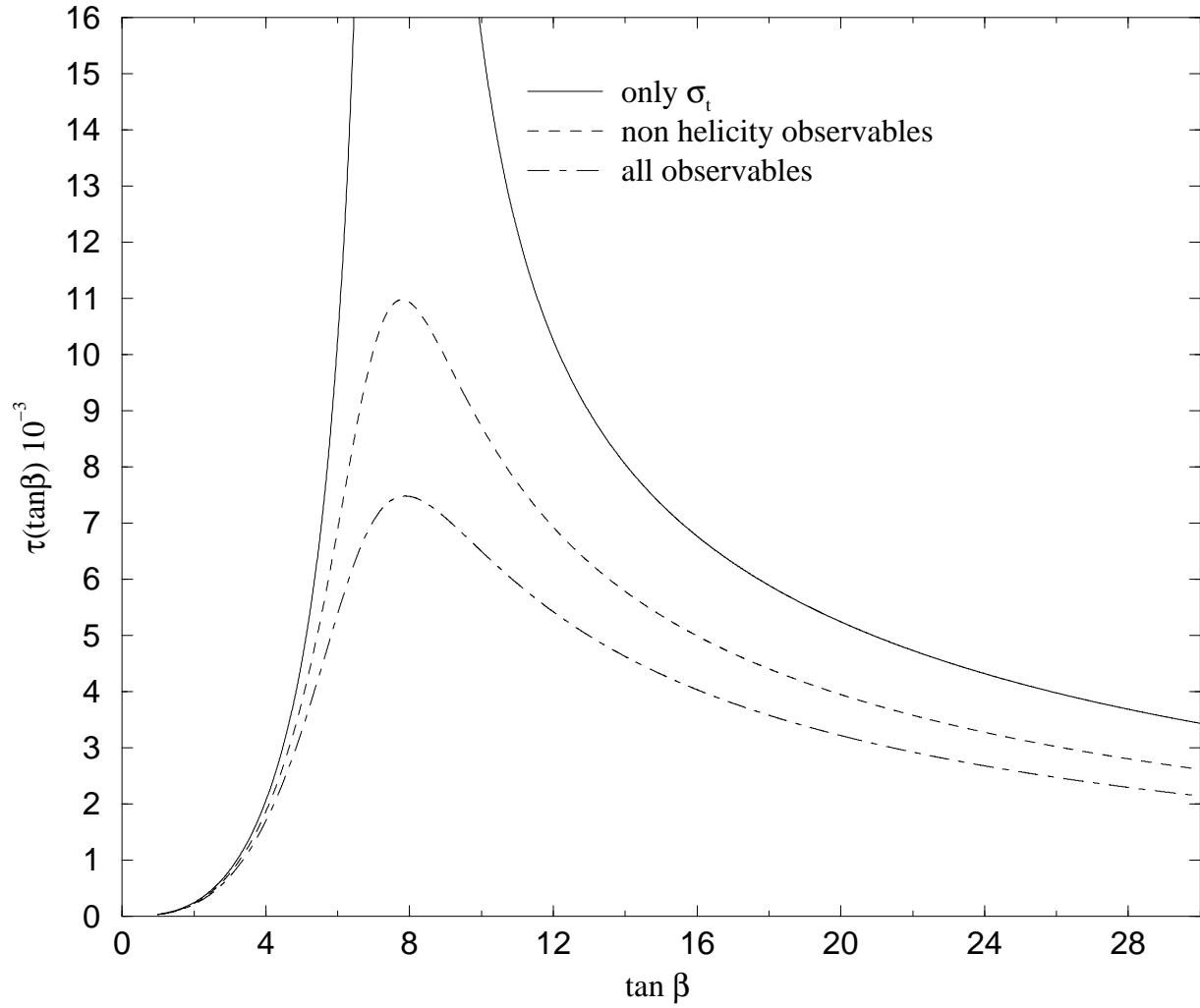


FIG. 4. Plot of the function  $\tau(\tan \beta)$  when the set of observables combined for the analysis is (i)  $\sigma_t$  alone, (ii)  $\sigma_t$ ,  $A_{FB,t}$ ,  $A_{LR,t}$  and  $A_t$ , (iii) the previous four and also the three helicity observables  $H_t$ ,  $H_{FB,t}$  and  $H_t^{LR}$ .

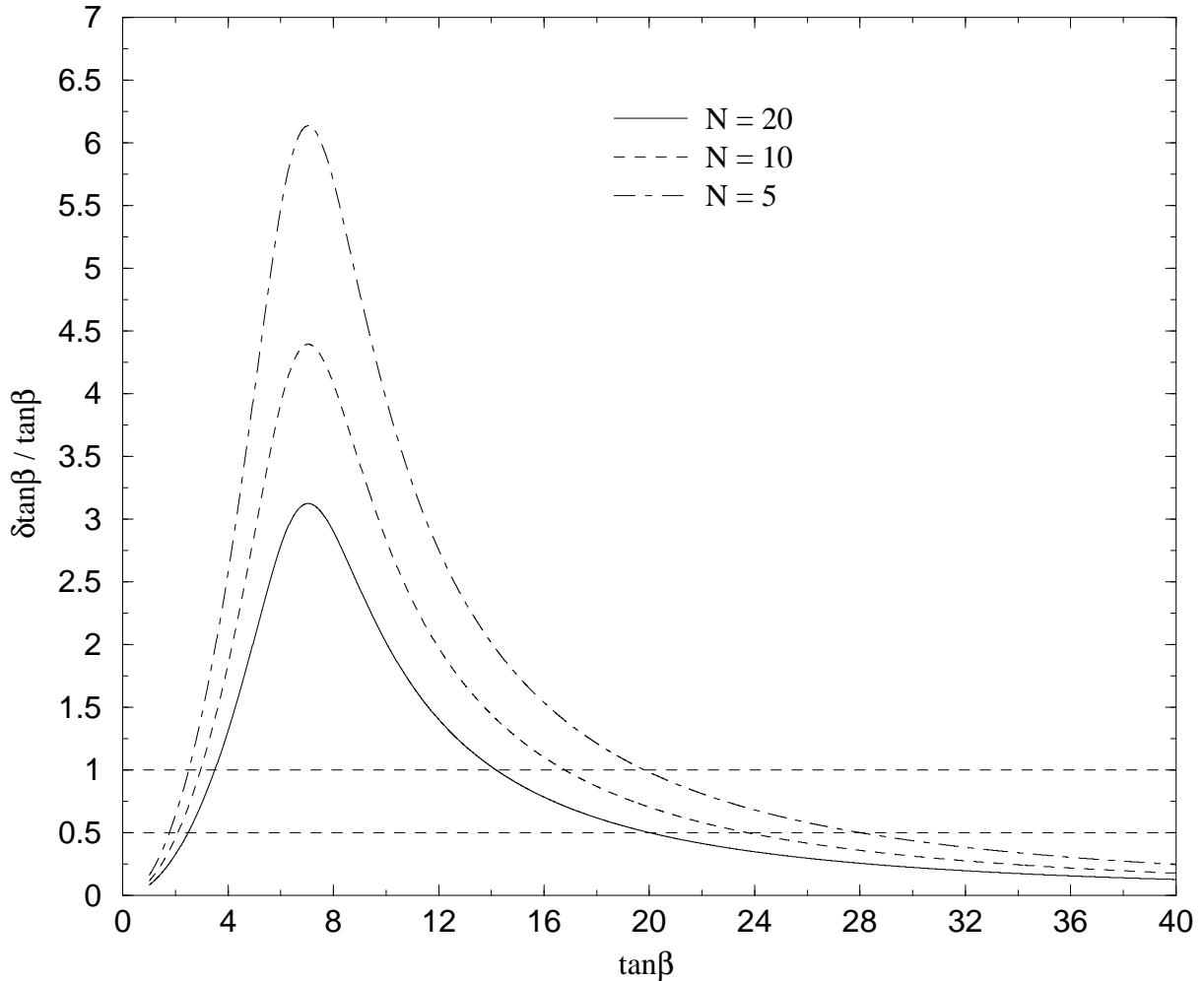


FIG. 5. Plot of the relative error on  $\tan\beta$ . The statistical accuracy is  $\sigma = 1\%$  for all the observables.  $N$  is the number of c.m. energy values at which independent measurements are taken. The curves actually depend on the combination  $\sigma/\sqrt{N}$ . One of the dashed lines corresponds to a 100 % relative error. With  $N = 10$  measurements, it determines a region  $3 < \tan\beta < 17$  where the determination of  $\tan\beta$  is completely unsatisfactory due to the flatness of the coefficient of the SUSY Sudakov logarithms with respect to  $\tan\beta$ . The second line marks the 50% accuracy level and identify the region  $\tan\beta < 2$  or  $\tan\beta > 20$ .